Linear systems, row reduction and echelon forms: Part 2

We now illustrate the row reduction algorithm to find the reduced echelon form of any matrix and how this allows for the solving of linear systems.

Example: Solve the following linear system:

$$\begin{aligned} 3x_2 - 6x_3 + 4x_4 &= -5 \\ 3x_1 - 7x_2 + 8x_3 + 8x_4 &= 9 \\ 3x_1 - 9x_2 + 12x_3 + 6x_4 &= 15 \end{aligned}$$

Step 0 Write the augmented matrix for the system.

Step 1 Begin with the leftmost column (this is a pivot column). Select a nonzero entry in this column as a pivot and, if necessary, interchange rows to move this entry into the pivot position.

Step 2 Use elementary row operations to create zeros in all positions below the pivot.

Step 3 Find the leftmost column with a nonzero leading entry in a row not yet containing a pivot. (This is the next pivot column.) Apply steps 1 and 2 to this column.

Step 4 Repeat steps 1 - 3 until there are no more nonzero rows to modify.

Step 5 Beginning with the rightmost pivot and working upward and to the left, use elementary row operations to create zeros above each pivot.

Step 6 Use scaling operation to make each pivot 1 and write down the corresponding system of equations.

Step 7 Identify free and basic variables and give a parametric description of the solution set.